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Re-engineering Investment Management

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Abstract

The conventional approach to building portfolios of investments is inefficient. The usual hierarchy of investment decisions imposes constraints on active management which are, under most circumstances, unnecessary and produce mean-variance inefficient portfolios. We propose a new approach to portfolio design, which eliminates these harmful constraints. It is based on the concept of alpha portability, and also relies on an analogous idea, beta portability. We present our position both conceptually and mathematically, and we illustrate the benefit of our approach with numerical examples. Our analysis reveals that alpha independence and separation are necessary but insufficient conditions for mean-variance efficiency. These features must be combined with leverage to effect portability and achieve the best expected outcome.

Re-engineering Investment Management[1]

The conventional approach to portfolio design starts with determining how much capital to invest in each of several asset classes. This is called the asset allocation decision. Some investors implement this allocation passively, by investing in index funds.[2] Other investors try to pick superior active managers to gain exposure to various asset classes. They instruct each active manager to outperform a passive benchmark portfolio, which is chosen to represent the asset class's performance. At the same time, the active managers are expected to maintain reasonable tracking fidelity to their benchmarks. Most investors employ some combination of passive and active management to implement their asset allocation.

For those investors who employ active management, there are potentially two sources of return and risk in the portfolio. One is passive return, which is the compensation for bearing the systematic risks embedded in each asset class. Then there is active return, or alpha, which is expected return earned without bearing systematic risk. Alpha is the return to the assumed superior skill of the active manager.

Asset allocation starts by partitioning the universe of investment opportunities into discrete asset classes. Portfolio weights are based on the asset classes' expected returns, standard deviations, and correlations. A benchmark portfolio typically represents each asset class. We can think of the asset allocation decision as selecting the weights of a portfolio of these passive benchmark portfolios. We call this portfolio of exposures to passive benchmarks the investor's beta portfolio; it is also called the policy portfolio.

The second source of risk and return is the away-from-benchmark positions taken by the active managers on behalf of the investor. Because each of these active positions is designed to add alpha, we call each active manager's portfolio of overweight and

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[1] We would like to thank Peter Bernstein, Craig Emrick, Frank Fabozzi, Jack Meyer, and participants at the Q Group 2004 Spring Seminar and the 2004 State Street Associates Research Retreat for helpful comments.

[2] Other instruments available to give passive exposure to broad asset classes include stock and bond futures contracts, and total return swaps. Unlike index funds, these instruments offer the flexibility of providing exposure without requiring much capital. Misused, that flexibility can be explosive; however, the existence of instruments that give simple and essentially capital-free access to broad asset classes is important in making possible the re-engineering we propose. Twenty years ago, when such instruments were less common, our proposal would have been impractical.

underweight positions the alpha portfolio. In other words, a manager's alpha portfolio is the difference between the actual portfolio and the benchmark portfolio. When we combine all the active managers' alpha-seeking positions, we call the result the investor's alpha portfolio.

Therefore, we can think of a portfolio as two sub portfolios: an assortment of benchmark portfolios, generally chosen by the investor, sometimes with the assistance of a consultant, called the investor's beta portfolio; and an assortment of active portfolios chosen by the investor's investment managers, and collectively called the investor's alpha portfolio. Any investment portfolio can be decomposed this way into an alpha portfolio and a beta portfolio.

Suppose we were to optimize the structure of the beta portfolio, optimize the structure of the alpha portfolio, and then combine the efficient alpha and beta portfolios. Would the result be an efficient portfolio? In general, the combination of two mean-variance efficient sub-portfolios is not, itself, a mean-variance efficient portfolio. The exception is when the two sub-portfolios' returns are independent. The alpha and beta portfolios are independent.[3] That means portfolio construction is modular. In order to construct a portfolio we start by building two risk-return efficient sub-portfolios, the alpha portfolio and the beta portfolio. Then we size each of these sub-portfolios to reflect the return and risk each offers and combine them.[4]

[3] Note that by definition, alpha is expected return that is not associated with bearing systematic risks. So alpha is independent of systematic risk factors. Of course, that does not mean that active managers do not try to assume systematic risks, sometimes in artfully subtle ways, and inaccurately label the resulting return as alpha.

[4] We size the alpha and beta portfolios so that their marginal information ratios are equal. This result relies on the independence of the alpha and beta portfolios.

I. Remembrance of Things Past

We can formally derive an efficient portfolio by maximizing expected utility, which is defined by equation (1):[5]

$$E(U) = \mu_p - \lambda_{RA}\sigma_p^2, \tag{1}$$

where,

$E(U)$ = expected utility,
μ_p = portfolio expected return,
λ_{RA} = a parameter representing risk aversion, and
σ_p^2 = portfolio variance.

If we solve this problem without borrowing restrictions, it is well known that the solution is the sum of a risk free asset or equivalently, a hedging portfolio which defeases liabilities, and a return-seeking speculative portfolio.[6] An investor who is perfectly risk averse would select the defeasing portfolio. The speculative portfolio trades off expected return for risk. It is a self-financing (zero-net-wealth) portfolio, and its structure depends on the investor's aversion to risk, the expected returns of the assets, and their standard deviations and correlations.

Our interest is in the structure of the speculative portfolio.[7] We wish to partition it. Recall that there are two potential sources of expected return for the investor to tap, exposure to risky asset classes (the beta portfolio) and exposure to the vagaries of active managers' skills (the alpha portfolio). Further, we can think of the beta portfolio as a

[5]Equation (1) assumes investors have quadratic utility and therefore care only about mean and variance. Financial economists typically assume that power utility is a more plausible description of investor preferences. For a wide range of returns, however, we can use mean and variance to approximate power utility. See, for example, Levy Markowitz (1979).

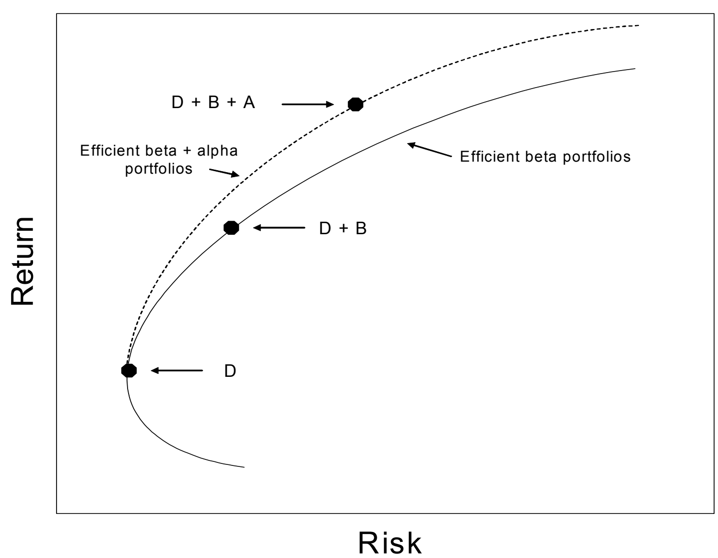
[6] The assumption of no borrowing restrictions is important. We return to it in Section III. For a derivation of this result, see Thomas (1985).

[7] The defeasing portfolio is important but not interesting in our context. It is, in principal, different for every investor. For example, when considering a young investor investing for his eventual retirement, and who is concerned with the real value of his portfolio in the distant future, the defeasing portfolio would probably be dominated by long maturity TIPS. Many investors would want to place some of their wealth in more aggressive investments, like common stocks. The speculative portfolio describes this part of the investor's holdings.

portfolio of benchmarks, each representing a single asset class. The alpha portfolio combines active portfolios contributed by each of the active managers. Suppose n is the number of asset classes into which the investor divided the universe of securities when he performed his asset allocation, and m is the number of active managers the investor has hired. Then the speculative portfolio contains n benchmark sub-portfolios, labeled $b_1 \ldots b_n$, and m active sub-portfolios, labeled $a_1 \ldots a_m$. In total, there are n + m return-seeking bets taken by the investor, n of them on benchmark portfolios and m of them on alpha portfolios.

Figure 1 shows the risk-return tradeoff associated with different portfolio compositions. Portfolio D is the risk free portfolio which defeases liabilities. The region to the right of the solid line represents the collection of portfolios accessible using passive management only, and those on the solid line to the right of portfolio D are efficient. This region represents policy portfolios, which we call beta portfolios. D + B represents the result of assuming exposure to the optimal beta portfolio. Imagine an indifference curve that is tangent to the collection of feasible policy portfolios at D + B. The investor selects D + B in preference to D because he hopes to earn a higher return. Recall that B is constructed to be an efficient portfolio of systematic risks embedded in asset class benchmarks.

Figure 1



If the investor believes active managers can add value, then the investor can use them to access portfolios with superior risk-return characteristics – those along the dashed line.[8] The investor confronted by an augmented set of investment opportunities, based on manager skill, will prefer a result such as D + B + A to D + B. Again, the carrot is extra prospective return, the stick is extra risk, and you should imagine an indifference curve tangent to the augmented efficient frontier at point D + B + A.

The investor's optimal portfolio, D + B + A, consists of the defeasing portfolio (the investor's risk free asset) plus return seeking beta and alpha bets. The condition for any bet to enter the investor's portfolio is the same, whether it is a beta bet or an alpha bet. For each bet, the ratio of marginal expected return divided by marginal contribution to risk must be the same. Further, for each bet the ratio of marginal return to marginal risk must be equal to the investor's degree of risk aversion.

All bets, whether they are beta bets or alpha bets, are created equally. Think of it this way: when you optimize a portfolio, the optimizer does not know whether a particular bet is a beta bet or an alpha bet. It knows only the bet's expected return and its contribution (on the margin) to the overall portfolio's volatility, based on the bet's standard deviations and correlations with all other bets.

In conventional practice, however, the way we construct portfolios implies that alpha and beta bets are not created equally. Beta trumps alpha. Conventionally, investors make the asset allocation decision first. As a result, the policy portfolio – the portfolio of beta bets – is optimized to be mean-variance efficient. Unfortunately, that means alpha bets get short changed. Having designed an optimal beta portfolio, investors subsequently can only optimize the alpha portfolio subject to any restrictions that arise from the structure of the beta portfolio. For example, if a particular asset class is excluded from the beta portfolio, then the alpha portfolio will exclude any active positions that might have been taken by managers who specialize in this particular asset class. No matter how skillful those specialist managers are, their potential contributions will not enter the investor's portfolio. These restrictions, produced by the conventional hierarchy of

[8] Obviously, all investors cannot use above-average skill to access superior risk-return opportunities. For each investor who uses active management to beat the opportunity set offered to passive-only portfolios, there is another investor who will find that his active positions have produced an inferior outcome.

investment decisions – asset allocation first, active positions second -- mean the alpha portfolio is unlikely to be mean-variance efficient.[9]

II. Anticipation of Things to Come

We wish to argue that you can eat your cake and have it, too – sometimes. To eliminate the need to sacrifice the alpha portfolio in order to optimize the beta portfolio, investors should allocate a portfolio's capital to the risk free asset, and then introduce both the alpha and beta portfolios as overlays to it. The alpha and beta portfolios could then be constructed independently, so both could be efficient portfolios.

In this section we illustrate the comparative statics of our approach incrementally, using simple mathematics to trace the progression from conventional portfolio design to our proposed design. We want to show what is necessary and what is sufficient, to effect different elements of the portfolio design we propose. We focus on the design of the speculative portfolio.

We begin with the simplest case, by considering a mean-variance investor who wishes to construct only a beta portfolio of stocks and bonds, both passively managed.[10] The second case substitutes an actively managed stock fund for the passive stock fund. The third case disentangles the actively managed stock fund into its beta component and its pure alpha component, and lets the investor allocate across passive bonds, the beta component of the stock fund, and the alpha component of the stock fund, fully collateralized by the risk free asset. Fourth, we assume that the investor can borrow in order to finance the alpha portfolio. Borrowing means the investor does not need to shift capital away from passive stocks and bonds in order to gain exposure to the alpha portfolio. Fifth and last, we assume the investor can borrow to finance both the alpha portfolio and the beta portfolio. Under these circumstances we treat both the alpha

[9] Moreover, in practice the composition of the alpha portfolio is compromised in another important way. Active positions are taken by different active managers, acting independently. This decentralized approach to constructing the alpha portfolio means that one manager's active positions are not coordinated with the active positions taken by other managers. In general, this is sub optimal. These concerns are beyond the scope of this paper. See Sharpe (1981) for discussion of the implications of decentralized investment management.

[10] See, for example, Markowitz (1952).

portfolio and the beta portfolio as overlays to a position in the risk free asset, which absorbs all of the portfolio's capital. We therefore present the following progression: 1) passive management, 2) bundled active management, 3) unbundled active management, 4) alpha portability, and 5) alpha and beta portability.

Case 1: The Optimal Mix with Passive Stocks and Bonds

In case 1, we wish to choose a portfolio comprising stocks and bonds, passively managed, so that we maximize expected utility as defined by equation (1). We assume that borrowing and lending are not permitted.

In order to identify the allocations that maximize expected utility, we re-write equation (1) as a function of the investor's exposure to stocks and bonds. The portfolio's expected return is simply a linear combination of each asset's expected return, as shown by equation (2).

$$\mu_p = w_S \mu_S + w_B \mu_B \tag{2}$$

where,
μ_p = portfolio expected return
μ_S = expected return of stocks
μ_B = expected return of bonds
w_S = percentage of the portfolio allocated to stocks
w_B = percentage allocated to bonds

Portfolio variance, however, depends on the variance of stocks and bonds as well as their covariance, as shown by equation (3):

$$\sigma_p^2 = w_S^2 \sigma_S^2 + w_B^2 \sigma_B^2 + 2 w_S w_B \rho \sigma_S \sigma_B \tag{3}$$

where,
σ_p^2 = portfolio variance
σ_S = standard deviation of stocks
σ_B = standard deviation of bonds
ρ = correlation between stocks and bonds

Equation (4) restates equation (1) as a function of the stock and bond weights.

$$E(U) = w_S\mu_S + w_B\mu_B - \lambda_{RA}\left(w_S^2\sigma_S^2 + w_B^2\sigma_B^2 + 2\rho w_S w_B \sigma_S \sigma_B\right) \tag{4}$$

In order to determine the optimal weights (those that maximize expected utility), we first measure how sensitive expected utility is to changes in exposure to each asset by taking the partial derivative of expected utility with respect to each asset's weight.

$$\frac{\partial E(U)}{\partial w_S} = \mu_S - \lambda_{RA}\left(2w_S\sigma_S^2 + 2w_B\rho\sigma_S\sigma_B\right) = 0 \tag{5}$$

$$\frac{\partial E(U)}{\partial w_B} = \mu_B - \lambda_{RA}\left(2w_B\sigma_B^2 + 2w_S\rho\sigma_S\sigma_B\right) = 0 \tag{6}$$

These derivatives represent the marginal utility of each asset, and they measure how much we increase or decrease expected utility, starting from the current asset mix, by increasing exposure to that asset. A negative partial derivative indicates that we improve expected utility by reducing exposure to that asset, while a positive partial derivative indicates that we should raise the exposure to that asset in order to improve expected utility.

Suppose we estimate the expected returns and standard deviations of stocks and bonds, passively managed, and the correlation between them, from the hypothetical passive returns shown in Table 1.

Table 1: Hypothetical Passive Returns		
Year	Passive Stocks	Passive Bonds
1	-24.70%	-0.83%
2	-0.49%	7.84%
3	22.21%	8.68%
4	9.86%	-1.86%
5	40.33%	2.47%
6	32.34%	13.19%
7	8.36%	17.79%
8	-22.67%	-19.05%
9	1.35%	8.39%
10	13.42%	13.36%
Return	8.00%	5.00%
Risk	20.00%	10.00%
Correlation		0.54

Furthermore, suppose we wish to choose a beta portfolio for an investor whose risk aversion equals 1.00, and that the investor's portfolio is currently allocated 100% to stocks and 0% to bonds. Risk aversion of 1.00 means an investor is willing to reduce expected return by one unit in order to lower variance by one unit.

If we substitute the expected returns, standard deviations, and the correlation from Table 1, along with our assumption about risk aversion, into the equations (5) and (6), we find that bonds have a marginal utility of 0.028 compared to 0.000 for stocks. To raise expected utility, we should therefore increase exposure to bonds. Because the marginal utility of stocks is 0.00, we would not increase marginal utility by changing the stock exposure. However, we cannot increase exposure to bonds without shifting funds away from stocks. We should therefore increase exposure to bonds because they have a higher derivative than stocks and reduce by the same amount exposure to stocks. In this way, we ensure that we are always 100% invested. (Remember, we disallow borrowing and lending in this case.)

Suppose we shift 1% of our portfolio from stocks to bonds. Bonds still have a higher derivative than stocks (0.028 versus 0.001); hence we again shift funds from stocks to bonds. If we proceed in this fashion, we find that when the portfolio is allocated equally to stocks and bonds, the stock and bond derivatives are exactly equal to each other at 0.029. At this allocation, we cannot improve expected utility by shifting between stocks and bonds. We have maximized expected utility, which equals 4.71%.[11] The expected return of the portfolio equals 6.50%, and its standard deviation equals 13.38%.[12]

Case 2: The Optimal Mix with Actively Managed Stocks

Now suppose we identify an active stock manager who we are confident will add value compared to passive stocks. We base this conviction on the hypothetical active stock returns shown in Table 2.

Table 2: Hypothetical Active and Passive Returns

Year	Active Stocks	Passive Bonds
1	-22.15%	-0.83%
2	-3.45%	7.84%
3	22.07%	8.68%
4	12.08%	-1.86%
5	42.82%	2.47%
6	31.40%	13.19%
7	9.10%	17.79%
8	-23.05%	-19.05%
9	5.35%	8.39%
10	15.86%	13.36%
Expected return	9.00%	5.00%
Standard deviation	20.10%	10.00%
Correlation		0.54

This actively managed fund has a 1.00% higher expected return than passive stocks, while its standard deviation is only 0.10% higher. The correlation between active

[11] William F. Sharpe introduced this marvelously intuitive algorithm for portfolio optimization. For more detail, see Sharpe (1987).
[12] An equally weighted portfolio of stocks and bonds maximizes this investor's expected utility. It would be natural to suppose that an equally weighted portfolio must also be mean-variance efficient. In fact, it is easy to show that a mean-variance efficient portfolio combines stocks and bonds in the ratio of 19:81, not 50:50. The investor is induced to own an equally weighted portfolio instead because he is not permitted to use leverage, so the only way for him to take more risk is to buy more of the riskier asset, stocks. A levered, 19:81 portfolio of stocks and bonds could produce an expected return of 7% for the same risk as the 50:50 portfolio; it would invest 102% of the investor's capital in passive bonds, and 24% in passive stocks.

stocks and passive bonds is slightly less than the correlation between passive stocks and bonds, but it is not apparent when rounded to two decimal places.

If we substitute the expected return, standard deviation, and correlation based on this actively managed stock portfolio into equations (5) and (6), we find that the partial derivatives are no longer equal when the portfolio is allocated equally to stocks and bonds. The stock derivative has increased to 0.039 while the bond derivative remains equal to 0.029. In order to maximize expected utility, we must therefore shift funds from bonds to stocks until their derivatives are again equal, which occurs when we allocate 67% to stocks and 33% to bonds. By shifting from a passive to active stock fund we raise expected utility from 4.71% to 5.28%. Moreover, expected return increases from 6.50% to 7.67%, while the portfolio's standard deviation increases from 13.38% to 15.46%.

Case 3: The Optimal Mix with a Pure Alpha Portfolio

In case 2, the investor has access to a bundled stock fund comprising passive and active components. The passive component represents a pure exposure to systematic stock risks. The active stock fund mixes exposure to passive stocks and the investment manager's skill. In other words, it mixes beta exposures and alpha exposures. Suppose instead that the investor is confronted with unbundled exposures to these two components: a pure exposure to passive stocks, and a pure exposure to the investment manager's skill. How would our solution differ?

To produce a pure alpha exposure, purged of all exposures to asset class returns, we could purchase the active stock fund and sell the stock benchmark short. The alpha portfolio's returns would equal the difference between the active stock returns and the passive stock returns. Table 3 shows that the pure alpha portfolio has an expected return of 1.00% and a standard deviation of 2.00%.

Table 3: Hypothetical Active, Passive, and Alpha Returns				
Year	Passive Stocks	Passive Bonds	Active Stocks	Alpha Portfolio
1	-24.70%	-0.83%	-22.15%	2.55%
2	-0.49%	7.84%	-3.45%	-2.97%
3	22.21%	8.68%	22.07%	-0.14%
4	9.86%	-1.86%	12.08%	2.22%
5	40.33%	2.47%	42.82%	2.49%
6	32.34%	13.19%	31.40%	-0.94%
7	8.36%	17.79%	9.10%	0.75%
8	-22.67%	-19.05%	-23.05%	-0.39%
9	1.35%	8.39%	5.35%	3.99%
10	13.42%	13.36%	15.86%	2.44%
Return	8.00%	5.00%	9.00%	1.00%
Risk	20.00%	10.00%	20.10%	2.00%

Table 4 shows the correlations between each pair of assets. Notice that the returns of the alpha portfolio are uncorrelated with passive stocks and bonds.

Table 4: Correlations				
	Passive Stocks	Passive Bonds	Active Stocks	Alpha Portfolio
Passive Stocks	1.00	0.54	1.00	0.00
Passive Bonds	0.54	1.00	0.54	0.00
Active Stocks	1.00	0.54	1.00	0.10
Alpha Portfolio	0.00	0.00	0.10	1.00

Next we fully collateralize the alpha portfolio with a risk free asset that has a 3.00% return. The collateralized alpha portfolio therefore has an expected return of 4.00%. Its standard deviation and correlations remain 2.00% and 0.00%. Expected utility is now a function of three assets: passive stocks, passive bonds, and the collateralized alpha portfolio, as shown in equation (7).

$$E(U) = w_S \mu_S + w_B \mu_B + w_A \mu_A - \lambda_{RA}\left(w_S^2 \sigma_S^2 + w_B^2 \sigma_B^2 + w_A^2 \sigma_A^2 + 2\rho w_S w_B \sigma_S \sigma_B\right) \qquad (7)$$

where,

μ_A = expected return of the active portfolio

σ_A = standard deviation of the active portfolio

Because the returns of the collateralized alpha portfolio are independent of the returns of the passive components, the derivatives of expected utility with respect to passive stocks and bonds remain unchanged. The derivative with respect to the alpha portfolio is given by equation (8).

$$\frac{\partial E(U)}{\partial w_A} = \mu_A - \lambda_{RA}\left(2 w_A \sigma_A^2\right) = 0 \tag{8}$$

To determine the utility-maximizing allocation to these three assets, we again compute the derivatives and continually shift funds from the asset with the lowest derivative to the asset with the highest derivative. If we start from the optimal passive weights of 50% stocks and 50% bonds, we find that the marginal utility of the collateralized alpha portfolio is 0.040 compared to 0.029 for passive stocks and bonds. Therefore, we shift funds from the passive components to the collateralized alpha portfolio. If we disallow short selling, we find that we maximize expected utility when we allocate 50% to passive stocks, nothing to passive bonds, and 50% to the collateralized alpha portfolio. At these allocations the portfolio's expected utility falls to 4.97%, and expected return and standard deviation fall to 6.02% and 10.15%.[13]

Disentangling the alpha component and introducing it as a separate, collateralized asset does not improve expected utility even though it is independent of passive stocks and bonds. Indeed, the disentangled alpha portfolio is barely more attractive than passive stocks, and it serves as only a slightly superior substitute for passive bonds.

What's going on? The unleveraged alpha portfolio does not offer a very high expected return compared to passive stocks. Therefore, an investor who is relatively willing to accept risk in order to increase expected return is not particularly enamored of an asset that has a 4.00% expected return and a 2.00% standard deviation. Even though the return to risk ratio of the collateralized alpha portfolio is greater than the return to risk ratio of passive stocks, and even though it is uncorrelated with the passive assets, it is not

[13] If we allow short positions but not leverage, such that we restrict the portfolio's exposure to any combination of assets to 100% of the portfolio's capital, the optimal weights call for a 3% short exposure to passive bonds, a 51% exposure to passive stocks, and a 52% exposure to the collateralized active portfolio, which produces expected utility of 4.99%.

very interesting unless it is leverageable. Without using leverage, it may not be possible to produce a portfolio that efficiently combines alpha exposures with beta exposures.

Case 4: The Optimal Mix with Alpha as an Overlay

Suppose instead that the investor introduces the alpha portfolio to the portfolio without using capital from the passive funds. For example, the investor might sell futures contracts that track passive stocks in order to finance the alpha portfolio. Alternatively, the investor might finance the long positions in the alpha portfolio with offsetting short positions. The critical point is that the investor does not need to shift capital from passive stocks and bonds to a risk free asset in order to collateralize the alpha portfolio. How much exposure to the alpha portfolio is optimal now that it is leverageable?

The math provides the intuition. In this case the derivative of the alpha portfolio need not equal the derivatives of the passive components to maximize expected utility; rather it merely needs to be positive. (Remember, it no longer needs to compete with the passive funds for capital.) Remarkably, the optimal alpha portfolio exposure rises from 52% when capital is required to 1,250% when capital is not required. At this allocation, along with a 50/50 mix of passive stocks and bonds, expected utility equals 10.96%, based on an expected return of 19.00% and a standard deviation of 28.35%.

Case 5: The Optimal Mix with Alpha and Beta as Overlays

Finally, the apotheosis of financial engineering: we allocate all of the portfolio's capital to the risk free asset, which has a 3.00% return, and we treat passive stocks, passive bonds, and the unadulterated alpha portfolio as overlays, and as infinitely leverageable overlays, at that. The optimal alpha exposure remains unchanged at 1,250%, because it is uncorrelated with passive stocks and bonds. The 50/50 mix between passive stocks and bonds shifts to 46% passive stocks and 200% passive bonds. Note that both passive bonds and the alpha portfolio have the same ratio of return to risk (0.50), yet the alpha portfolio is more than six times as attractive because it is uncorrelated with passive stocks. This comparison illustrates the value of independence.

The Comparative Statics of Portfolio Design

By incrementally progressing from the conventional portfolio design to our proposed design, we observe the impact of relaxing the constraints of the conventional approach. Table 5 summarizes these comparative statics.

Table 5: Comparative Statics of Active Management (λ = 1.00)

	Passive Stocks	Passive Bonds	Active Stocks	Alpha Portable	Riskless Asset	Expected Return	Standard Deviation	Expected Utility
Passive Stocks Passive Bonds	50%	50%				6.50%	13.38%	4.71%
Active Stocks Passive Bonds		33%	67%			7.67%	15.46%	5.28%
Alpha Portfolio Unlevered	50%	0%		50%	50%	6.02%	10.15%	4.97%
Alpha Portfolio Levered	50%	50%		1250%		19.00%	28.35%	10.96%
Alpha & Beta Portfolios Levered	46%	200%		1250%	100%	29.19%	36.19%	16.10%

Table 5 yields three important insights:

1. Disentangling the alpha component from the beta component, in and of itself, does not raise expected utility if we collateralize the alpha portfolio with a risk free asset that draws capital away from passive stocks and bonds.
2. Independence is important. If we allow unlimited exposure to both passive bonds and the alpha portfolio, which have identical return to risk ratios, the uncorrelated alpha portfolio is six times as attractive as correlated passive bonds.
3. The greatest impact, however, arises from leveraging the alpha and beta portfolios it in a way that does not draw capital away from passive stocks and bonds. For example, the alpha portfolio is 25 times as attractive when it does not absorb capital; its optimal allocation rises from 50% to 1,250%!

These results, of course, are specific to the assumptions used in these examples. Nonetheless, these assumptions are reasonably illustrative of conventional views. We would be remiss, however, if we failed to highlight the sensitivity of these results to the most critical and least defensible assumption -- that alpha is positive. If the true expected return of the alpha portfolio were -1% rather than +1%, the optimal allocation to the alpha portfolio would shift from +1,250% to -1,250%! By comparison, if the alpha component remained bundled with the passive component, and if alpha were really -1% rather than +1%, the optimal allocation to actively managed stocks would be 32% rather than 67% -- which serves as a rather stark illustration of the distinction between volatility and uncertainty.

III. Summary and Conclusion

In conventional practice, the most important decision an investor makes is the asset allocation decision. It is the first decision; typically, the decision rests on the tradeoff between risk and prospective return, with stocks playing the role of the relatively risky asset, and bonds the lower risk choice.[14] Other asset classes, such as foreign stocks and bonds, private equity, real estate, and commodities, may also receive allocations. Then there are decisions to be made within each asset class: to use passive management, active management, or a combination of the two; and, which managers to select. As we have observed, the composition of the active portfolio is seldom explicitly considered in the traditional approach to managing investments. Neither is the attribution of risk between what we call beta risk and alpha risk. The traditional approach can be expected to produce mean-variance inefficient portfolios. In particular, the active components of the portfolio are never explicitly optimized, but fall out as a by product of the asset allocation and manager selection decisions.

We propose a different approach. First, we argue that the principal objective is to defease liabilities; hence investors should first identify the liabilities, and then construct a portfolio that minimizes risk with respect to those liabilities. Investors should, before all else, purchase this defeasing portfolio.

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[14] Already, we should sense that something is wrong. To increase risk, one changes the portfolio allocation between stocks and bonds. A key insight of the CAPM is that the optimal way to modulate a portfolio's risk is by changing the mix of risky and risk assets rather than the composition of the portfolio of risky assets.

For some investors that is the end of the story. However, others may wish to earn a higher expected return than that which is offered by the defeasing portfolio. They must accept some risk to do so. For these investors, the next step is to decide how much risk, in total, they wish to bear. That is the risk budget, and the remaining steps are designed to produce the greatest expected return that is possible, without bearing more than this much risk.

The next step could be to sell a portion of the defeasing portfolio, in order to fund the return seeking positions. That is not the approach we advocate. For investors seeking more return, at higher risk, we propose instead adding two overlay portfolios: the beta portfolio and the alpha portfolio.[15] The beta portfolio produces expected return by exposing the investor to systematic risks. The alpha portfolio produces expected return by exposing the investor to active positions selected by skilled investment managers. Because the alpha and beta portfolios are independent, each can be assembled separately, and the components then can be combined.

Practical Considerations

We have presented our approach conceptually, assuming implicitly that it is practically feasible. We would now like to explore some practical considerations of our proposed portfolio design.

The Defeasing Portfolio

The defeasing portfolio is specific to each investor. Ideally, it should generate cash flows that match the liabilities of the investor. As a first approximation, we could choose a fixed income portfolio whose cash flows match the investor's liabilities. This portfolio would comprise cash instruments as well as interest rate swaps. This would work fine if the liabilities were known with certainty and fixed through time, but this is not always the case. First of all, investors may suffer a reduction in other sources of income, which would place a greater burden on the fund to defease liabilities. Moreover,

[15] An overlay portfolio is a self financing portfolio. Accordingly, decisions to use overlay portfolios do not require disturbing the defeasing portfolio. One of the objectives is to avoid the need to sell defeasing assets in order to hold return seeking assets.

liabilities typically grow, at least with the pace of inflation, since investors strive to preserve the purchasing power of their disbursements. Finally, fixed income instruments do not provide constant cash flows forever. Bonds mature, and the repayments must be reinvested at uncertain interest rates. As a consequence of the uncertain nature of both the liabilities and the income stream of the defeasing assets, we must estimate their growth rates, standard deviations, and correlations in order to identify a collection of assets that will best defease liabilities, which in almost all circumstances will involve some degree of risk.

The Beta Portfolio

For many risky asset classes such as stocks and bonds, constructing a self financing beta portfolio and attaching it to the defeasing portfolio – beta portability - is straightforward owing to the widespread availability of futures contracts and total return swaps. Some risky asset classes, notably real estate and private equity, are less easy to replicate with derivatives. We suggest that the investor deploy capital to access these exposures and employ financing wherever in the portfolio it is efficient to do so.

The Alpha Portfolio

As we define the alpha portfolio, it has two essential features: it is independent of the defeasing portfolio and the beta portfolio; and it is self financing and hence infinitely leverageable. When measured empirically, the alpha portable may display non-zero correlations with the defeasing or beta portfolio, because there will typically be sampling error. However, the expected correlations of the alpha portfolio should be zero. If the alpha portfolio systematically has non-zero correlations and the manager is unwilling to offset these systematic risks, then the investor could offset them by engaging in futures or swaps transactions or simply by redesigning the beta portfolio.

The self financing assumption may pose a more significant challenge, because it is not always possible to finance long alpha trades with short alpha trades or even with short beta trades that hedge out the systematic risks of the long alpha trades. Nor is it always possible to implement alpha trades with derivatives. As an alternative, the investor could finance the defeasing portfolio and use the available capital to create the alpha portfolio. Again, the investor should finance wherever inexpensive financing is available, and reserve capital for the components that are not easily financed.

Finally, it is important to bear in mind that the alpha and beta portfolios should each be mean-variance efficient, and their marginal information ratios should be equal.

Conclusion

Modern portfolio theory is fifty years old. Nevertheless, some of its insights are not reflected in the conventional construction of investment portfolios. In particular, the active portions of portfolios are almost certainly inefficient, and the balance of risk between alpha and beta exposures is not efficiently struck.

The implications of these arguments are profound. If the suggestions we make for re-engineering portfolio construction are adopted, the structure of the money management industry would change substantially. We would expect asset management to evolve in ways that favor absolute return managers at the expense of core managers. Investors in the future would probably assume more alpha risk, and less beta risk, than they conventionally do today. The value added by active managers – or the lack of it – would be more transparent, so that active managers would face more competitive pressures to produce alpha than currently they do. Moreover, these pressures would be exacerbated because alpha portability expands the competitive landscape: all active managers would be in the same business, creating alpha at an attractive information ratio. Hence, they would all compete with all other active managers, rather than just other active managers in their own asset class.

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